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Event-Triggered H_∞ Filter Design of T-S Fuzzy Systems Subject to Hybrid Attacks and Sensor Saturation

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ABSTRACT This paper addresses the problem of event-triggered H_∞ filter design for nonlinear systems under both hybrid attacks and sensor saturation. A novel event-triggered mechanism (ETM) is proposed to reduce the number of transmission data per unit time, thereby improving the network QoS and taking the control performance into account. Considering that the deception attack may deteriorate the control performance, ETM is designed to be sensitive to deception attacks, that is, the average data release rate during the deception attack is higher than other periods. Consequently, the control performance can be improved. Moreover, the proposed ETM can reduce the occurrence of erroneous triggering events that are aroused by abnormal abrupt variation. In addition, the denial-of-service (DoS) attack is considered as well. A switching filtering error model is established based on the fact of each period of DoS attack being classified as active period and sleeping period. In addition, the problem of measurement saturation is concerned in filter design. By using Lyapunov-Krasovskii stability theory, sufficient conditions are obtained to guarantee the stability of the fuzzy filtering error system. The effectiveness of filter design methods is finally verified by a simulation example.

INDEX TERMS Event-triggered mechanism, hybrid attacks, sensor saturation, filter design.

I. INTRODUCTION

Nonlinear systems are commonly and widely exist in the real-world [1]–[3], such as vehicle systems, social medical systems, etc. Takagi-Sugeno (T-S) fuzzy model is an effective method to model nonlinear systems, and has been widely used in filtering and control design of nonlinear systems [4]–[6]. The nonlinear system, under the T-S fuzzy model, is represented by a series of linear subsystems associated with membership functions. For example, in [7], suspension systems was described by a T-S fuzzy model with IF-THEN rules. For fuzzy-based networked nonlinear control systems, the premise variables are transmitted over the network. The asynchronous problem should be taken into account. In [8], some constrains to the premise variables were introduced to investigate such a problem.

Periodical sampling and releasing may waste the limited communication and computation resources since the updated period is determined in consideration of the worst case, which may lead to some conservatism of the control system and a decrease in the network quality of service (QoS). Event-triggered mechanism (ETM) is a promising alternative, since the data released into the network is event-driven rather than a time-driven. The data-releasing event only occurs at the instant when the control system needs. Consequently, the quantity of the data releasing per unit time is largely reduced, thereby reducing the burden of the network bandwidth. Thanks to this advantage, recently, ETM has received extensive attention from both theoretical and practical perspectives [9]–[11]. An event-triggered scheme for output-based leader-following consensus was designed for a class of nonlinear multi-agent systems in [12]. The authors investigated the problem of both the quantization and the ETM to further relieve the burden of network transmission in [13], for T-S fuzzy -based filtering systems. For the purpose of improving the control/estimation performance, the adaptive ETMs were studied in [14], [15]. It is noted that erroneous triggering events may happen when the measurement suffering from

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abnormal variation. These data are redundant for filter, which will occupy the network resource. However, few results focus on this issue, which motives our current research.

Network transmission for filtering systems may induce the problem of cyber-attacks, which is a crucial challenge to the estimation [16], [17]. Deception attacks and denial of service (DoS) attacks are adopted by the malicious adversaries commonly [13], [17], [18]. For example, In [19], a resilient control strategy against periodic DoS attacks was investigated by converting a networked control system (NCS) into a switched system. The occurrence of DoS attacks were modeled by a Markov process in [18], under which a resilient control strategy was proposed. The authors in [20] used historical information to investigate the detection of deception attacks in cyber-physical systems (CPSs). Taking both deception attacks and quantization phenomenon into account, the authors discussed the distributed filtering problem for a class of discrete-time system in [21]. The authors studied a resilient output feedback control for networked interconnected systems against deception attack in [22], [23]. As mentioned above, only single attack mode has been discussed in the control system in some existing literature. In practice, the malicious attackers usually uses a switched attack mode on a control system to achieve the purpose of disordering the control systems. However, few results concerning with switched attacks on the nonlinear filtering systems are available, which is another motivation of this study.

The problem of sensor saturation may happen when the amplitude of the measurement beyond a certain level, which will degrade the estimation performance of the filter. In view of the significance of sensor saturation, many results have been obtained from researchers. For example, the problem of sampled data approach to H_∞ filtering was presented in [24] for neural network subject to sensor saturation that satisfies sector conditions. The saturation of both the actuator and the sensor was considered in [25], under which the authors addressed the problem of dynamic output feedback control for discrete-time Markov jump linear systems. In [26], the problem of H_∞ control was investigated for time-delay systems under simultaneous consideration of missing measurement, channel fading and sensor saturation.

In this paper, we deal with the problem of event-triggered H_∞ secure filtering for T-S fuzzy-model-based nonlinear systems with sensor saturation and hybrid attacks. The main contributions of the study are highlighted as follows: (1) A new ETM is introduced. Compared to the existing traditional ETMs, the proposed ETM is more sensitive to deception attacks, especially to random deception attacks. The ETM will generate much more data-releasing events when suffering form random deception attacks than other periods. The capacity of the system against cyber-attacks is thus improved. However, the average data releasing rate remains a lower lever comparing with the time-triggered mechanism. In addition, the number of erroneous triggering events can be reduced by introducing average value in the ETM. (2) Hybrid attacks and sensor saturation are simultaneously considered. To get a prescribed estimation performance of nonlinear systems subject to DoS attacks and sensor saturation, a tolerant filtering design approach is proposed by converting the system into a switched T-S fuzzy-model-based system. Based on such a model, a new resilient security criteria is put forward with consideration of the proposed ETM.

The remainder framework of this study is arranged as follows. Problem formulation of the resilient secure T-S fuzzy-based filtering for nonlinear systems against hybrid cyber-attacks and sensor saturation is described in Section II. Section III presents filter design method to ensure the exponential stability of nonlinear systems. Numerical results are given in Section IV to demonstrate the effectiveness of the proposed design method. Section V summarizes the full paper.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of real $n\times m$ matrices respectively. $|x|$ represents the absolute value of x, X^T and X^{-1} represent the transpose and inverse of matrix X respectively. $\|\cdot\|$ stands for the Euclidean norm of a vector. The notation $X > 0$ (respectively, $X < 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, negative definite). $\mathbb{E}\{V\}$ stands for the expectation of stochastic variable V. The asterisk $*$ in a matrix denotes the term that is induced by symmetry of a matrix.

II. PROBLEM FORMULATION

A. MODEL CONSTRUCTION

Consider a nonlinear system that can be represented by a T-S fuzzy model with m plant rules as follows:

Plant Rule i: IF $g_1(t)$ is $\mathcal{G}_{i1}, \dots, g_r(t)$ is $\mathcal{G}_{ir}$, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i\omega(t) \\ z(t) = E_i x(t) \\ y(t) = C_i x(t) \end{cases} \tag{1}$$

where $\mathcal{G}_{i1}, \mathcal{G}_{i2}, \cdots, \mathcal{G}_{ir}$ are the fuzzy sets ($i = 1, 2, \cdots, m$), $g_1(t)$, $g_2(t)\dots$, $g_r(t)$ are the premise variables. $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $y(t) \in \mathbb{R}^{n_y}$ is the measured output, $\omega(t) \in l_2[0,\infty)$ is the exogenous disturbance signal, $z(t) \in \mathbb{R}^{n_z}$ represents the output to be estimated. A_i, B_i, C_i and E_i are known real matrices with appropriate dimensions.

By applying the technology of center-average defuzzifier, product interference and singleton fuzzifier, the global dynamics model of the T-S fuzzy system (1) can be inferred as:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{m} \theta_i(g(t))\left[A_i x(t) + B_i\omega(t)\right] \\ z(t) = \sum_{i=1}^{m} \theta_i(g(t)) E_i x(t) \\ y(t) = \sum_{i=1}^{m} \theta_i(g(t)) C_i x(t) \end{cases} \tag{2}$$

where the membership function is denoted by $\theta_i(g(t)) = \frac{\varsigma_i(g(t))}{\sum_{i=1}^{m}\varsigma_i(g(t))} \geq 0$ and $\sum_{i=1}^{m}\theta_i(g(t)) = 1$, the premise variables $g(t) = [g_1(t), g_2(t), \cdots, g_r(t)]^T$ and $\varsigma_i(g(t)) = \Pi_{s=1}^{r}\mathcal{G}_{is}(g_s(t))$.

Denote the measurement output $y(t) \triangleq [y^1(t), y^2(t), \cdots, y^{n_y}(t)]$. Taking sensor saturation into account, we define

$$sat(y^i(t)) = \begin{cases} \bar{y}^i, & y_i > \bar{y}^i \\ y^i(t), & -\bar{y}^i \leq y_i(t) \leq \bar{y}^i \\ -\bar{y}^i, & y_i < \bar{y}^i \end{cases} \tag{3}$$

where $sat(\cdot)$ is the saturation function, and $\bar{y}^i(t)$ denotes the known saturation upper bound.

Similar to [27], the measurement output in (2) with sensor saturation is considered as

$$\check{y}(t) = sat(y(t)) = y(t) - \varrho(y(t)) \tag{4}$$

where nonlinear function $\varrho(y(t))$ satisfies

$$\varrho^T(y(t))\varrho(y(t)) \leq \sigma y^T(t)y(t) \tag{5}$$

for a scalar $\sigma \in (0, 1)$.

For the sake of network communication, the premise variable between the plant and the fuzzy filter is asynchronous, here, we denote the new premise variables as $\hat{g}(t)$. Then the following filter form is exploited to estimate $z(t)$.

Plant Rule j: IF $\hat{g}_1(t)$ is $\mathcal{G}_{j1},\ldots, \hat{g}_r(t)$ is $\mathcal{G}_{jr}$, THEN

$$\begin{cases} \dot{x}_f(t) = A_{fj}x_f(t) + B_{fj}y_f(t) \\ z_f(t) = C_{fj}x_f(t) \end{cases} \tag{6}$$

where $x_f(t) \in \mathbb{R}^{n_x}$ and $z_f(t) \in \mathbb{R}^{n_z}$ is the filter state vector and the output of the filter respectively, A_{fj}, B_{fj} and C_{fj} are the filter coefficient matrices need to be designed.

Similarly, we can obtain the overall fuzzy filter dynamic as follows

$$\begin{cases} \dot{x}_f(t) = \sum_{j=1}^{m}\theta_j(\hat{g}(t))[A_{fj}x_f(t) + B_{fj}y_f(t)] \\ z_f(t) = \sum_{j=1}^{m}\theta_j(\hat{g}(t))C_{fj}x_f(t) \end{cases} \tag{7}$$

For the simplification of expression, $\theta_i(g(t))$ and $\theta_j(\hat{g}(t))$ will be abbreviated as θ_i^g and $\theta_j^{\hat{g}}$, respectively in the following. Also, an assumption to the membership function is made by $\theta_j^{\hat{g}} - \iota_j\theta_j^g \geqslant 0$ $(0 < \iota_j \leq 1)$.

B. EVENT-TRIGGERED MECHANISM

For a purpose of saving limited communication bandwidth, an ETM is introduced between the sensor and the filter to determine whether the sampling data should be sent. Here, we suppose the sampler is time-driven with a fixed period h. The releasing instant is denoted by t_kh with $\{t_k\}_{k=0}^{\infty}$ being a monotonically increasing sequence of integers. The latest releasing data is denoted by $\check{y}(t_kh)$. Then the current sampling can be indicated by $\check{y}(t_kh + lh)$, $l = 1, 2, 3, \cdots$.

For $t \in [t_kh, t_{k+1}h)$, we define

$$\begin{aligned} \hbar(t_{k,l}) &= \alpha[\check{y}(t_kh + lh) - \check{y}(t_kh)] + \check{y}(t_kh) \\ e_k(t) &= \check{y}(t_kh) - \hbar(t_{k,l}) \\ \mathscr{F}(t) &= \frac{\mu_2}{2}\left[\check{y}^T(t_kh)\Omega e_k(t) + e_k^T(t)\Omega\check{y}(t_kh)\right] \end{aligned} \tag{8}$$

where α is a scalar with $\alpha \in (0, 1]$, μ_1, μ_2 are given weight scalar.

Then we construct the following ETM to determine the next releasing instant

$$\zeta(t) = e_k^T(t)\Omega e_k(t) - \mu_1\check{y}^T(t_kh)\Omega\check{y}(t_kh) + \mathscr{F}(t) \leq 0 \tag{9}$$

where Ω is a weighting matrix, $\mu_1 > 0$, $\mu_2 > 0$ are given positive scalars.

The releasing event is triggered only when the condition (9) is violated, that is, the next releasing instant is determined by

$$t_{k+1}h = t_kh + h + \max\{lh|\zeta(t) < 0\} \tag{10}$$

Remark 1: In (9), the main purpose of using $\hbar(t_{k,l})$ for calculating $e_k(t)$ to replace $\check{y}(t_kh + lh)$ which is used in the traditional ETM is to smooth the input signal. Specially, if one takes $\alpha = 1$ and $\mu_2 = 0$, the ETM will degrade to a traditional one as in [28], [29], If α takes 0.5, $\hbar(t_{k,l})$ becomes the average of $\check{y}(t_kh + lh)$ and $\check{y}(t_kh)$. Compared to the traditional ETM, the ETM with the definition of $e_k(t)$ in (8) will generate less erroneous events induced by the abrupt variation of the output measurement.

Remark 2: The parameters α and $\mu_i (i = 1, 2)$ in (8) are the weight factor. If α approaches to 1, $\hbar(t_{k,l})$ tends to the current sampling value, while α is near 0, $\hbar(t_{k,l})$ is around the latest sampling value. The lager the value of μ_2, the more sensitive it is to the disturbance and deception attacks. To choose a proper value of μ_2, one should make a trade-off between this sensitivity and the average data-releasing rate to in the design of ETM.

C. DECEPTION ATTACKS

Malicious adversaries launch deception attacks by injecting the attack signals into the transmission signal to achieve the purpose of degrade the control performance and even to destroy the control system.

Motivated by [30], we assume the attack signal $\delta(t)$ described by a nonlinear function with the following constrain

$$\| \delta(t) \|_2 \leq \| Fx(t) \|_2 \tag{11}$$

where F is a known matrix.

Taking the sensor saturation and deception attack into account, we obtain the filter input as

$$\tilde{y}_f(t) = \check{y}(t) + \delta(t) \tag{12}$$

Remark 3: Although a bigger magnitude of deception attack is, the greater destruction to the system, the malicious adversaries usually restrain the attack signal with respective to the transmission signal in the light of the following reasons:

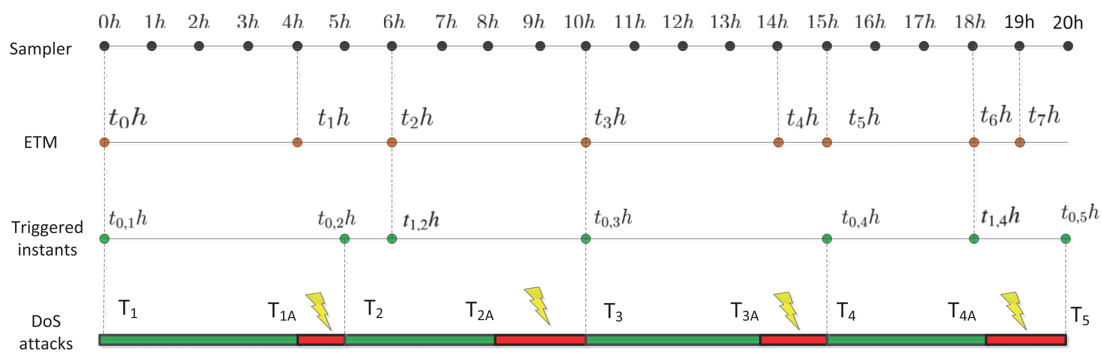


FIGURE 1. The sequence of DoS attacks and ETM.

1) to avoid being detected; 2) to decrease limited energy consumption.

Remark 4: To enhance the estimation performance of the system against deception attacks, the item $\mathscr{F}(t)$ is introduced in the ETM that we proposed in (9). By this design, the average data releasing rate during deception attacks is higher than other periods, since the ETM is more sensitive to the deception attacks than the traditional design of ETMs, which will be demonstrated in Section IV.

D. DoS ATTACKS

DoS attack is another typical cyber-attack. Different from deception attacks, the way of DoS attack is no longer to load attack signal into the transmission signal, but to block the signal successful transmission. The filter will fail to estimate the output of the system if zero input of the filter exceeds the maximum duration.

A full DoS attack with a fixed period T is divided into two parts: sleeping period and active period. We denote a-th DoS attack time interval, sleeping time interval and active time interval by $\mathscr{D}_a$, $\mathscr{S}_a$, and $\mathscr{A}_a$, respectively. As is shown in Figure 1, T_a is the start time of the a-th DoS attack period. Then we have $\mathscr{S}_a = [T_a, T_a + S_a)$, $\mathscr{A}_a = [T_a + S_a, T_{a+1})$. Consequently, $\mathscr{D}_a = \mathscr{S}_a \cup \mathscr{A}_a = [T_a, T_{a+1})$, $T = T_{a+1} - T_a$ and $0 < S_a < T$. For convenience, we denote $T_{aA} \triangleq T_a + S_a$ in the following. Then $T_d = T_{a+1} - T_{aA}$ denotes the duration time of DoS active period.

To have a further analysis to the secure filtering system, we make the following assumptions

Assumption 1: ETM is neglected when the sampling data at the end of the attack period, that is, the sampling data at T_{a+1} should be released into the network so as to strengthen the reliability of the system.

Assumption 2: The output measurement is set to zero if it is transmitted over the network during active period of DoS attack, otherwise, it can be successfully transmitted.

Based on the above assumptions together with the ETM, we have

$$\hat{y}_f(t) = \begin{cases} \check{y}(t_{w,a}h), & t \in [t_{k,a}h, t_{k+1,a}h) \cap \mathscr{S}_a \\ 0, & t \in \mathscr{A}_a \end{cases} \tag{13}$$

where $t_{w,a}h$ represents the w-th instant that the triggering event is generated by ETM during the a-th sleeping period, $(w = 0, 1, 2, \cdots, \bar{w})$, at which the packet can be successfully transmitted over the network.

Remark 5: Based on the Assumption 1, it has

$$\{t_{w,a}h\} = \{T_a\} \cup \{t_k h\} \tag{14}$$

when the set $\{t_{w,a}h\}$ extends to the real releasing instant sequence.

E. HYBRID ATTACKS

In a practical situation, the deception attack and DoS attack are usually alternated. To characterize this kind of hybrid attack behavior, we first consider a case that a random deception attack launches on the signal $\check{y}(t_{k,a}h)$ in (4) during the sleeping period. It follows that

$$\bar{y}_f(t) = \check{y}(t_{w,a}h) + \beta(t)\delta(t_{w,a}h) \tag{15}$$

for $t \in [t_{k,a}h, t_{k+1,a}h) \cap \mathscr{S}_a$, where $\beta(t)$ is a random variable with $\beta(t) \in \{0, 1\}$ and the expectation being $\mathbb{E}\{\beta(t)\} = \overline{\beta}$.

Taking Assumption 2 into account, one can know that the real filter input can be represented by

$$y_f(t) = \begin{cases} \bar{y}_f(t_{w,a}h), & t \in [t_{k,a}h, t_{k+1,a}h) \cap \mathscr{S}_a \\ 0, & t \in \mathscr{A}_a \end{cases} \tag{16}$$

For technical convenience, we define

$$\bar{\Upsilon}^l_{w,a} = [t_{w,a}h + lh, t_{w,a}h + lh + h) \tag{17}$$

where $l = 0, 1, 2, \cdots, \bar{l})$, $\bar{l} = \min\{t_{w+1,a}, t_{0,a+1}\} - t_{w,a} - 1$.

Let $\Upsilon^{l,I}_{w,a} = \bar{\Upsilon}^l_{w,a} \cap \mathscr{A}_a$ and $\Upsilon^{l,II}_{w,a} = \bar{\Upsilon}^l_{w,a} \cap \mathscr{S}_a$.

For $t \in \Upsilon^{l,II}_{w,a}$, the error $e_k(t)$ in (8) then need to be redefined as

$$e_{w,a}(t) = \check{y}(t_{w,a}h) - \check{y}(t_{w,a}h + \Delta h) \tag{18}$$

where $\check{y}(t_{w,a}h + \Delta h) = \alpha[\check{y}(t_{w,a}h + lh) - \check{y}(t_{w,a}h)] + \check{y}(t_{w,a}h)$.

Defining $d_{w,a}(t) = t - t_{w,a}h - lh$ for $t \in \Upsilon^{l,II}_{w,a}$, we can rewrite the filter input during the DoS attack sleeping period is

$$\begin{aligned} y_f(t) = \sum_{i=1}^{m} \theta_i(\hat{g}(t))[\frac{1}{\alpha}e_{w,a}(t) + C_i x(t - d_{w,a}(t)) \\ + \beta(t)\delta(t - d_{w,a}(t)) - \varrho(C_i x(t - d_{w,a}(t)))] \end{aligned} \tag{19}$$

from (16).

Define $\xi(t) = [x^T(t), x_f^T(t)]^T$ and $\mathcal{E}(t) = z(t) - z_f(t)$.

Combining (2), (7) and (19), and taking Assumption 2 into account, we can know that the filter system can be converted to a switched system with the follow two mode

$$I: \begin{cases} \dot{\xi}(t) = \sum_{i=1}^{m}\sum_{j=1}^{m} \theta_i^g \theta_j^{\hat{g}} [\tilde{A}_{ij}\xi(t) + \tilde{E}_i\omega(t)] \\ \mathcal{E}(t) = \sum_{i=1}^{m}\sum_{j=1}^{m} \theta_i^g \theta_j^{\hat{g}} \tilde{G}_{ij}\xi(t) \end{cases} \quad (20)$$

for $t \in \mathscr{A}_a$, which represents the system is in DoS active period; and

$$II: \begin{cases} \dot{\xi}(t) = \sum_{i=1}^{m}\sum_{j=1}^{m} \theta_i^g \theta_j^{\hat{g}} [\tilde{A}_{ij}\xi(t) + \tilde{B}_j e_{w,a}(t) \\ +\tilde{C}_{ij}H\xi(t - d_{w,a}(t)) + \tilde{D}_j\delta(t - d_{w,a}(t)) \\ +\tilde{E}_i\omega(t) + \tilde{F}_j\varrho(C_i x(t - d_{w,a}(t)))] \\ \mathcal{E}(t) = \sum_{i=1}^{m}\sum_{j=1}^{m} \theta_i^g \theta_j^{\hat{g}} \tilde{G}_{ij}\xi(t) \end{cases} \quad (21)$$

for $t \in \mathscr{S}_a$, which depicts the system in DoS sleeping period, where $\tilde{A}_{ij} = \begin{bmatrix} A_i & 0 \\ 0 & A_{fj} \end{bmatrix}$, $\tilde{B}_j = \begin{bmatrix} 0 \\ \frac{1}{\alpha}B_{fj} \end{bmatrix}$, $\tilde{C}_{ij} = \begin{bmatrix} 0 \\ B_{fj}C_i \end{bmatrix}$, $\tilde{D}_j = \begin{bmatrix} 0 \\ \beta(t)B_{fj} \end{bmatrix}$, $\tilde{E}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}$, $\tilde{F}_j = \begin{bmatrix} 0 \\ -B_{fj} \end{bmatrix}$, $\tilde{G}_{ij} = \begin{bmatrix} E_i & -C_{fj} \end{bmatrix}$, $H = \begin{bmatrix} I & 0 \end{bmatrix}$.

In this paper, we intend to design the novel ETM in (9) for the networked fuzzy filter (7), such that the nonlinear system (2) with sensor saturation is exponentially stable with H_∞ performance level γ under the above hybrid attack. The design objective is summarized as the following two aspects:

(i) The filtering error system (20) and (21) with $\omega(t) = 0$ are exponentially stable in mean square sense.

(ii) Under zero initial condition, the inequality $\|\mathcal{E}(t)\|_2 < \gamma\|\omega(t)\|_2$ holds.

III. MAIN RESULTS

In this section, sufficient conditions for the exponential stability of the switched system with the mode of (20) and (21) will be derived by using Lyapunov-Krasocskii stability theory in Theorem 1, and then the filter design will be shown in Theorem 2 based on Theorem 1.

Theorem 1: Consider the constrains of sensor saturation in (4), deception attack in (11) and ETM in (9), the switched system with the mode I in (20) and II in (21) is H_∞ exponentially stable in means square sense with performance level γ, if for given positive scalars $\iota_i, \mu_1, \mu_2, \eta_n, \epsilon_n, \lambda_0, \lambda_2, \alpha, \sigma, \bar{\beta}$ with $n = 1, 2$, and matrices A_{fi}, B_{fi} and C_{fi}, and the maximum of DoS active period $T_d^{max} = \frac{\eta_1 T - \ln\sqrt{\lambda_0\lambda_2}}{(\eta_1+\eta_2)} - h$, there exists matrices $\Omega > 0, P_n > 0, Q_n > 0, R_n > 0$ and matrices Ψ_i with appropriate dimensions such that:

$$\Gamma_{ij} - \Psi_i < 0 \quad (22)$$

$$\iota_i(\Gamma_{ij} - \Psi_i) + \iota_j(\Gamma_{ij} - \Psi_j) + \Psi_i + \Psi_j < 0 (i \leq j) \quad (23)$$

$$P_{3-n} \leq \lambda_n P_n \quad (24)$$

$$R_n \leq \lambda_{4-2n} R_{3-n} \quad (25)$$

$$Q_n \leq \lambda_{4-2n} Q_{3-n} \quad (26)$$

for $i, j = 1, 2, \cdots m$, where

$$\Gamma_{ij} = \begin{bmatrix} \Gamma_{1n}^{ij} & * & * & * & * \\ \Gamma_{2n}^{ij} & -Q_n & * & * & * \\ \Gamma_{3n}^{ij} & 0 & -I & * & * \\ \Gamma_{4n} & 0 & 0 & -\Omega & * \\ \Gamma_{5n}^{i} & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\Gamma_{11}^{ij} = \begin{bmatrix} \Upsilon_{111}^{ij} & * & * & * & * & * & * \\ 0 & \Upsilon_{221} & * & * & * & * & * \\ \Upsilon_{311}^{ij} & \Upsilon_{321} & \Upsilon_{331} & * & * & * & * \\ \tilde{E}_i^T P_1 & 0 & 0 & -\gamma^2 I & * & * & * \\ \tilde{B}_j^T P_1 & 0 & 0 & 0 & -s_1\Omega & * & * \\ \tilde{D}_j^T P_1 & 0 & 0 & 0 & 0 & -I & * \\ \tilde{F}_j^T P_1 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\Gamma_{12}^{ij} = \begin{bmatrix} \Upsilon_{112}^{ij} & * & * & * & * \\ 0 & \Upsilon_{222} & * & * & * \\ \Upsilon_{312}^{ij} & \Upsilon_{322} & \Upsilon_{332} & * & * \\ \tilde{E}_i^T P_2 & 0 & 0 & -\gamma^2 I & * \\ \tilde{F}_i^T P_2 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\Upsilon_{11n}^{ij} = sym\left\{P_n\tilde{A}_{ij} + (-1)^{n+1}\eta_n P_n\right\} + H^T R_n H - \frac{\epsilon_n}{h}H^T Q_n H, \quad \Upsilon_{22n} = -\frac{\epsilon_n}{h}(hR_n + Q_n),$$

$$\Upsilon_{31n}^{ij} = (2-n)\tilde{C}_{ij}^T P_n + \frac{\epsilon_n}{h}Q_n H,$$

$$\Upsilon_{32n} = -\frac{\epsilon_n}{h}Q_n^T, \quad \Upsilon_{33n} = \sigma C_i^T C_i - \frac{2\epsilon_n}{h}Q_n,$$

$$\Gamma_{2n}^{ij} = \sqrt{h}Q_n H \begin{bmatrix} \tilde{A}_{ij} & 0 & \tilde{C}_{ij} & \tilde{E}_i & \tilde{B}_j & \tilde{D}_j & \tilde{F}_j \end{bmatrix},$$

$$\Gamma_{3n} = (2-n)\begin{bmatrix} 0 & 0 & F & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_{4n}^{i} = (2-n) \times \begin{bmatrix} 0 & 0 & s_2\Omega C_i & 0 & (s_3 + \frac{s_2}{\alpha})\Omega & 0 & -s_2\Omega \end{bmatrix},$$

$$\Gamma_{5n}^{ij} = \begin{bmatrix} \tilde{G}_{ij} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\epsilon_n = e^{-2\eta_n(2-n)h}, \quad \lambda_1 = \lambda_0 e^{2(\eta_1+\eta_2)h},$$

$$s_1 = 4\mu_1 + \mu_2^2, \quad s_2 = 2\mu_1, \quad s_3 = 0.5s_2 + \sqrt{s_1 - 2s_2}.$$

Proof: Choose the following piecewise Lyapunov-Krasovskii candidate as

$$V_n(t) = \xi^T(t)P_n\xi(t) + \int_{t-h}^{t} \wp_n \xi^T(s)H^T R_n H\xi(s)ds + \int_{-h}^{0}\int_{t+v}^{t} \wp_n \dot{\xi}^T(v)H^T Q_n H\dot{\xi}(v)dsdv$$

with $n = 1$ for the switched system subject to mode I in (20) and $n = 2$ for the switched system subject to the mode II in (21), where $\wp_n \triangleq e^{2(-1)^n\eta_n(t-s)}$.

First we considering the case that $n = 1$, taking derivation and expectation on $V_1(t)$ with respect to time $t \in \Upsilon^{l,I}_{w,a}$.

$$\begin{aligned}\mathbb{E}\{\dot V_1(t)\} \le \mathbb{E}\Big\{&-2\eta_1 V_1(t) + 2\eta_1 \xi^T(t)P_1\xi(t)\\ &+2\xi^T(t)P_1\dot\xi(t) + \xi^T(t)H^TR_1H\xi(t)\\ &+h\dot\xi^T(t)H^TQ_1H\dot\xi(t)\\ &-e^{-2\eta_1 h}\xi^T(t-h)H^TR_1H\xi(t-h)\\ &-e^{-2\eta_1 h}\int_{t-h}^{t}\dot\xi^T(s)H^TQ_1H\dot\xi(s)ds\Big\} \end{aligned} \tag{27}$$

By using Jensen inequality, we obtain

$$-\int_{t-h}^{t}\dot\xi^T(s)H^TQ_1H\dot\xi(s)ds \le \frac{1}{h}\bar\xi^T\begin{bmatrix}-Q_1 * * \\ Q_1 - 2Q_1 * \\ 0 Q_1 - Q_1\end{bmatrix}\bar\xi \tag{28}$$

where $\bar\xi = col\{H\xi(t), H\xi(t-d_{w,a}(t)), H\xi(t-h)\}$.

Recalling the constrain of deception attacks in (11), it follows that

$$\begin{aligned}&\delta^T(t-d_{w,a}(t))\delta(t-d_{w,a}(t))\\ &\quad\le \xi^T(t-d_{w,a}(t))H^TF^TFH\xi(t-d_{w,a}(t))\end{aligned} \tag{29}$$

From (4), we can get

$$\begin{aligned}&\varrho^T(C_iH\xi(t-d_{w,a}(t)))\varrho(C_iH\xi(t-d_{w,a}(t)))\\ &\quad\le \sigma(C_iH\xi(t-d_{w,a}(t)))^TC_iH\xi(t-d_{w,a}(t))\end{aligned} \tag{30}$$

Taking sensor saturation into account, the event triggering condition in (9) can be rewritten as

$$\begin{aligned}&s_1e^T_{w,a}(t)\Omega e_{w,a}(t)\\ &\quad\le [s_2\check y(t_{w,a}h) + s_3e_{w,a}(t)]^T\Omega[s_2\check y(t_{w,a}h) + s_3e_{w,a}(t)]\end{aligned} \tag{31}$$

Considering (28)-(31), we can get from (27) that

$$\begin{aligned}&\mathbb{E}\left\{\dot V_1(t) + 2\eta_1V_1(t) - \gamma^2\omega^T(t)\omega(t) + \mathcal{E}^T(t)\mathcal{E}(t)\right\}\\ &\qquad\le \sum_{i=1}^{m}\sum_{j=1}^{m}\theta_i^g\theta_j^{\hat g}\chi_1^T(t)\Gamma^{ij}\chi_1(t)\end{aligned} \tag{32}$$

where $\chi_1(t) = [\xi^T(t),\ \xi^T(t-h),\ \xi^T(t-d_{w,a}(t))H^T,\ \omega^T(t),\ e^T_{w,a}(t),\ \delta^T(t-d_{w,a}(t)),\ \varrho^T(y_i(t-d_{w,a}(t)))]^T$, $\chi_2(t) = [\xi^T(t),\ \xi^T(t-h)H^T,\ \xi^T(t-d_{w,a}(t))H^T,\ \omega^T(t),\ \varrho^T(y_i(t-d_{w,a}(t)))]^T$.

Inspired by the method in [31] to deal with the problem asynchronous premise variables, we can obtain

$$\begin{aligned}&\sum_{i=1}^{m}\sum_{j=1}^{m}\theta_i^g\theta_j^{\hat g}\chi_1^T(t)\Gamma_{ij}\chi_1(t)\\ &\quad\le \sum_{i=1}^{m}\sum_{j=1}^{m}\theta_i^g\theta_j^g\chi_1^T(t)\iota_j(\Gamma_{ij}-\Psi_i)\chi_1(t)\\ &\qquad+\sum_{i=1}^{m}\sum_{j=1}^{m}\theta_i^g\theta_j^g\chi_1^T(t)\Gamma_{ij}\chi_1(t)\\ &\quad\le \sum_{i=1}^{m}\theta_i^g\theta_i^g\chi_1^T(t)[\iota_i(\Gamma_{ii}-\Psi_i)+\Psi_i]\chi_1(t)\\ &\qquad+\sum_{i=1}^{m}\sum_{i<j}\theta_i^g\theta_i^g\chi_1^T(t)[\iota_j(\Gamma_{ij}-\Psi_i)+\iota_i(\Gamma_{ji}-\Psi_j)\\ &\qquad+\Psi_i+\Psi_j]\chi_1(t)\end{aligned} \tag{33}$$

from the assumption $\theta_j^{\hat g} - \iota_j\theta_j^g \geqslant 0$ $(0 < \iota_j \le 1)$ made in Section II-A.

It is obviously that (22) and (23) are the sufficient conditions to ensure $\sum_{i=1}^{m}\sum_{j=1}^{m}\theta_i^g\theta_j^{\hat g}\chi_1^T(t)\Gamma^{ij}\chi_1(t) < 0$ holds. Then it follows that

$$\mathbb{E}\{\dot V_1(t) + 2\eta_1V_1(t) + \mathcal{E}^T(t)\mathcal{E}(t) - \gamma^2\omega^T(t)\omega(t)\} \le 0 \tag{34}$$

For $n = 2$, the same method can be used to obtain

$$\mathbb{E}\{\dot V_2(t) - 2\eta_2V_2(t) + \mathcal{E}^T(t)\mathcal{E}(t) - \gamma^2\omega^T(t)\omega(t)\} \le 0 \tag{35}$$

First, we consider the design objective (i) in Section II. Let $\omega(t) = 0$ for (34) and (35), and we can obtain

$$\mathbb{E}\{V_n(t)\} \le \begin{cases} e^{-2\eta_1(t-T_a)}\mathbb{E}\{V_1(T_a)\}, & t \in \mathscr{S}_a\\ e^{2\eta_2(t-T_{aA})}\mathbb{E}\{V_2(T_{aA})\}, & t \in \mathscr{A}_a\end{cases} \tag{36}$$

Based on (24)-(26), one can obtain

$$\begin{cases}\mathbb{E}\{V_1(T_a)\} \le \lambda_2\mathbb{E}\{V_2(T_a^-)\}\\ \mathbb{E}\{V_2(T_{aA})\} \le \lambda_0\mathbb{E}\{V_1(T_{aA}^-)\}\end{cases} \tag{37}$$

A) For $t \in \mathscr{S}_a$, from (36) and (37), we can obtain

$$\begin{aligned}\mathbb{E}\{V(t)\} &\le \lambda_2e^{-2\eta_1(t-T_a)}\mathbb{E}\{V_2(T_a^-)\}\\ &\le \lambda_2e^{2\eta_2(T_a-T_{(a-1)A})-2\eta_1(t-T_a)}\mathbb{E}\{V_2(T_{(a-1)A})\}\\ &\le \lambda_0\lambda_2e^{\kappa(t)}\mathbb{E}\{V_1(T^-_{(a-1)A})\}\end{aligned}$$

where $\kappa(t) = 2(\eta_1+\eta_2)h + 2\eta_2(T_a - T_{(a-1)A}) - 2\eta_1(t-T_a)$.

Then

$$\mathbb{E}\{V(t)\} \le e^{-\phi(t)}\mathbb{E}\{V(0)\} \tag{38}$$

where $\phi(t) = 2\Big[\eta_1\sum_{i=0}^{a-1}S_a^i - \eta_2\sum_{i=0}^{a-1}T_d^i - a(\eta_1+\eta_2)h - a\ln\sqrt{\lambda_0\lambda_2} + \eta_1(t-T_a)\Big]$, $T_d = T_a - T_{(a-1)A}$ is the duration time of DoS active period.

Defining $\bar\phi = \eta_1 S_a^{\min} - \eta_2T_d^{\max} - (\eta_1+\eta_2)h - \ln\sqrt{\lambda_0\lambda_2}$ follows that $n\bar\phi \le \phi(t)$, which leads to

$$\mathbb{E}\{V(t)\} < e^{-2n\bar\phi}\mathbb{E}\{V(0)\} \tag{39}$$

from (38).

For $t \in \mathscr{S}_a$, it has $t + T_d \le (a+1)T$, where $T = T_{a+1} - T_a$ is the fixed DoS period, which is equivalent to

$$-a\bar\phi \le -\frac{\bar\phi}{T}t + \frac{(T-T_d)\bar\phi}{T} \tag{40}$$

Then, it follows that

$$\mathbb{E}\{V(t)\} \le \mathbb{E}\{V_1(0)\}e^{\frac{2\bar\phi(T-T_d)}{T}}e^{-\frac{2\bar\phi}{T}t} \tag{41}$$

B) For $t \in \mathscr{A}_a$, using the same method as above, we can get

$$\mathbb{E}\{V(t)\} \leq \frac{1}{\lambda_2}\mathbb{E}\{V_1(0)\}e^{-\frac{2\bar{\phi}}{T}t} \tag{42}$$

Integrating the case A) and the case B), we have

$$\mathbb{E}\{V(t)\} \leq \max\left\{\frac{1}{\lambda_2}, e^{\bar{\phi}(1-\frac{T_d}{T})}\right\}\mathbb{E}\{V_1(0)\}e^{-\frac{2\bar{\phi}}{T}t} \tag{43}$$

Define $g = \min\{\lambda_{\min}(P_n)\}$, $r = \max\left\{\lambda_{\max}(P_n) + h\lambda_{\max}(R_n) + \frac{h^2}{2}\lambda_{\max}(Q_n)\right\}$, where $\lambda(\cdot)$ denotes the eigenvalue. From the piecewise Lyapunov function, one can know that

$$\begin{aligned}\mathbb{E}\{V(t)\} &\geq g\|x(t)\|^2,\\ \mathbb{E}\{V_1(0)\} &\leq r\|\chi_1(0)\|_h^2\end{aligned} \tag{44}$$

Combining (43) and (44), one can obtain

$$\|x(t)\| \leq \sqrt{\frac{r}{g}\max\left\{\frac{1}{\lambda_2}, e^{\bar{\phi}(1-\frac{T_d}{T})}\right\}}\|\chi_1(0)\|_h e^{-\frac{\bar{\phi}}{T}t} \tag{45}$$

Then we can get the system (21) exponentially stable with the attenuation rate $\bar{\phi}/T$.

Next, we consider the design objective ii) in Section II that $\omega(t) \neq 0$. Under zero initial condition, for integrals on both sides of the inequality (34) and (35) from T_a to t, there is

$$\begin{aligned}\sum_{k=0}^{a}\int_{T_k}^{T_{k+1}}&[\mathbb{E}\{\dot{V}_n(t)\} - (-1)^n 2\eta_n\mathbb{E}\{V_n(t)\}\\ &+\mathcal{E}^T(t)\mathcal{E}(t) - \gamma^2\omega^T(t)\omega(t)]dt \leq 0\end{aligned} \tag{46}$$

Owning to $\mathbb{E}\{V_n(t)\} > 0$, we can obtain that $\sum_{k=0}^{a}\int_{T_k}^{T_{k+1}}[\mathcal{E}^T(t)\mathcal{E}(t) - \gamma^2\omega^T(t)\omega(t)]dt < 0$ from (36) with zero initial condition. It has

$$\int_{t_0}^{\infty}\|\mathcal{E}(t)\|^2 dt \leq \gamma^2\int_{t_0}^{\infty}\|\omega(t)\|^2 dt \tag{47}$$

for $a \to \infty$, which result in $\|\mathcal{E}(t)\|_2 < \gamma\|\omega(t)\|_2$.

Next, we will estimate the maximum duration of DoS active period to ensure the stability of the system with exponential attenuation.

From (33), one can know that $\bar{\phi}$ should be positive to guarantee the stability of the system, that is,

$$\eta_1 S_a^{\min} - \eta_2 T_d^{\max} - (\eta_1+\eta_2)h - \ln\sqrt{\lambda_0\lambda_2} > 0 \tag{48}$$

It is known that $S_a^{\min} + T_d^{\max} < T$, which leads to

$$T_d^{max} < \frac{\eta_1 T - \ln\sqrt{\lambda_0\lambda_2}}{(\eta_1+\eta_2)} - h \tag{49}$$

This completes the proof. ■

Through the above theorem, we have obtained sufficient conditions to make the system exponentially stable. In the next chapter, we will design the filter.

Theorem 2: Consider the constrains of sensor saturation in (4), deception attack in (11) and ETM in (9), the switched system with the mode I in (20) and II in (21) is H_∞ exponentially stable in means square sense with performance level γ, if for given positive scalars $\iota_i, \mu_1, \mu_2, \eta_n, \epsilon_n, \lambda_0, \lambda_2, \alpha, \sigma, \bar{\beta}$ with $n = 1, 2$, and the maximum of DoS active period $T_d^{max} = \frac{\eta_1 T - \ln\sqrt{\lambda_0\lambda_2}}{(\eta_1+\eta_2)} - h$, there exists matrices $P_{n1} > 0$, $\Omega > 0$, $Q_n > 0$, $R_n > 0$, and matrices $\tilde{\Psi}_i$, $\tilde{\Psi}_j$, Y_n, A_{fj}, B_{fj}, C_{fj} and matrices with appropriate dimensions such that:

$$\tilde{\Gamma}_{ij} - \tilde{\Psi}_i < 0 \tag{50}$$

$$\iota_i(\tilde{\Gamma}_{ij} - \tilde{\Psi}_i) + \iota_j(\tilde{\Gamma}_{ij} - \tilde{\Psi}_j) + \tilde{\Psi}_i + \tilde{\Psi}_j < 0 (i \leq j) \tag{51}$$

$$P_{n1} - Y_n > 0 \tag{52}$$

$$\begin{bmatrix} P_{(3-n)1} - \lambda_n P_{n1} & * \\ S_n - \lambda_n Y_n & L_n - \lambda_n Y_n \end{bmatrix} \leq 0, \tag{53}$$

$$R_n \leq \lambda_{4-2n}R_{3-n} \tag{54}$$

$$Q_n \leq \lambda_{4-2n}Q_{3-n} \tag{55}$$

$$\tilde{\Gamma}_{ij} = \begin{bmatrix} \tilde{\Gamma}_{1n}^{ij} & * & * & * & * \\ \tilde{\Gamma}_{2n}^{ij} & -Q_n & * & * & * \\ \tilde{\Gamma}_{3n}^{ij} & 0 & -I & * & * \\ \tilde{\Gamma}_{4n} & 0 & 0 & -\Omega & * \\ \tilde{\Gamma}_{5n}^{i} & 0 & 0 & 0 & -I \end{bmatrix} \tag{56}$$

where

$$\tilde{\Gamma}_{11}^{ij} = \begin{bmatrix} \tilde{\Upsilon}_{111}^{ij} & * & * & * & * & * & * & * \\ \Upsilon_{Y11}^{ij} & \Upsilon_{Y21}^{ij} & * & * & * & * & * & * \\ 0 & 0 & \Upsilon_{221} & * & * & * & * & * \\ \tilde{\Upsilon}_{311}^{ij} & C_i^T\hat{B}_{fj}^T & \Upsilon_{321} & \Upsilon_{331} & * & * & * & * \\ B_i^T P_{11} & B_i^T Y_1 & 0 & 0 & \Upsilon_{441} & * & * & * \\ \frac{\hat{B}_{fj}^T}{\alpha} & \frac{\hat{B}_{fj}^T}{\alpha} & 0 & 0 & 0 & \Upsilon_{551} & * & * \\ \bar{\beta}\hat{B}_{fj}^T & \bar{\beta}\hat{B}_{fj}^T & 0 & 0 & 0 & 0 & -I & * \\ -\hat{B}_{fj}^T & -\hat{B}_{fj}^T & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\tilde{\Gamma}_{12}^{ij} = \begin{bmatrix} \tilde{\Upsilon}_{112}^{ij} & * & * & * & * & * \\ \Upsilon_{Y12}^{ij} & \Upsilon_{Y22}^{ij} & * & * & * & * \\ 0 & 0 & \Upsilon_{222} & * & * & * \\ \tilde{\Upsilon}_{312}^{ij} & 0 & \Upsilon_{322} & \Upsilon_{332} & * & * \\ B_i^T P_{21} & B_i^T Y_2 & 0 & 0 & -\gamma^2 I & * \\ -\hat{B}_{fj}^T & -\hat{B}_{fj}^T & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\tilde{\Upsilon}_{11n}^{ij} = sym\left\{(-1)^{n+1}\eta_n P_{n1} + P_{n1}A_i\right\} + R_n - \frac{\epsilon_n}{h}Q_n,$$

$$\Upsilon_{Y1n}^{ij} = Y_n A_i + \hat{A}_{fj}^T + sym\{\eta_n Y_n\},$$

$$\Upsilon_{Y2n}^{ij} = sym\left\{\eta_n Y_n + \hat{A}_{fj}\right\},$$

$$\tilde{\Upsilon}_{31n}^{ij} = (2-n)C_i^T\hat{B}_{fj}^T + \frac{\epsilon_n}{h}Q_n,$$

$$\Upsilon_{441} = -\gamma^2 I, \Upsilon_{551} = -s_1\Omega,$$

$$\tilde{\Gamma}_{2n}^{ij} = \sqrt{h}Q_n\left[A_i \; 0 \; 0 \; 0 \; B_i \; 0 \; 0 \; 0\right],$$

$$\tilde{\Gamma}_{31} = \left[0 \; 0 \; 0 \; F \; 0 \; 0 \; 0 \; 0\right],$$

$$\tilde{\Gamma}^{i}_{41}\left[0\ 0\ 0\ s_2\Omega C_i\ 0\ (s_3+\tfrac{s_2}{\alpha})\Omega\ 0\ -s_2\Omega\right],$$
$$\tilde{\Gamma}^{ij}_{5n}=\left[E_i\ -\hat{C}^{T}_{fj}\ 0\ 0\ 0\ 0\ 0\ 0\right].$$

Moreover, the gains of filter are given by

$$\begin{cases} A_{fj}=P_{12}^{-1}\hat{A}_{fj}P_{12}^{-T}P_{13}\\ B_{fj}=P_{12}^{-1}\hat{B}_{fj}\\ C_{fj}=\hat{C}_{fj}P_{12}^{T}P_{13}\end{cases} \tag{57}$$

Proof: Define matrix $P_n=\begin{bmatrix}P_{n1} & P_{n2}\\ * & P_{n3}\end{bmatrix}>0$, $Y_n=P_{n2}P_{n3}^{-1}P_{n2}^{T}$ and $\Phi_n=\begin{bmatrix}I & 0\\ 0 & P_{n2}P_{n3}^{-1}\end{bmatrix}$.

Using Schur complement to $P_n>0$, we can obtain (52) holds. Through observing the inequality in (24) with $n=2$, we can get

$$\begin{bmatrix}P_{11}-\lambda_2P_{21} & *\\ P_{12}^{T}-\lambda_2P_{22}^{T} & P_{13}-\lambda_2P_{23}\end{bmatrix}\leq 0 \tag{58}$$

Define $S_2=P_{22}P_{23}^{-1}P_{12}^{T}$ and $L_2=P_{22}P_{23}^{-1}P_{13}P_{23}^{-1}P_{22}^{T}$. For $n=2$, pre- and post-multiplying (58) with $P_{22}P_{23}^{-1}$ and its transpose, it can be easily derived that (53) holds. For $n=1$, we can also get that (53) is equivalent to $P_2\leq\lambda_1P_1$.

Next, pre- and post-multiplying both sides of (22) and (23) by $diag\{\Phi_n, I, I, I, I, I\}$ and its transpose, calculate related items in theorem 1, theorem 2 can be obtained.

This completes the proof. ■

IV. A SIMULATION EXAMPLE

In this section, a numerical example of a T-S fuzzy-based nonlinear system is presented to verify the effectiveness of the proposed method. This nonlinear system is subjected to hybrid attacks and the saturation of the output measurement. The signal transmission from the plant to the filter is via wireless network.

Example 1: Consider the T-S fuzzy-model-based nonlinear system with the following parameters [32]:

$$A_1=\begin{bmatrix}-3 & 1 & 0\\ 0.3 & -2.5 & 1\\ -0.1 & 0.3 & -3.8\end{bmatrix},\quad B_1=\begin{bmatrix}1\\0\\1\end{bmatrix},$$
$$A_2=\begin{bmatrix}-2.5 & 0.5 & -0.1\\ 0.1 & -3.5 & 0.3\\ -0.1 & 1 & -2\end{bmatrix},\quad B_2=\begin{bmatrix}-0.6\\0.5\\0\end{bmatrix},$$
$$E_1=\begin{bmatrix}0.8 & 0.3 & 0\end{bmatrix},\quad E_2=\begin{bmatrix}-0.5 & 0.2 & 0.3\end{bmatrix},$$
$$C_1=\begin{bmatrix}0.5 & -0.1 & 1\end{bmatrix},\quad C_2=\begin{bmatrix}0 & 1 & 0.6\end{bmatrix}.$$

The membership functions are $\theta_1(g(t))=\sin^2(t)$ and $\theta_2(g(t))=\cos^2(t)$.

Suppose H_∞ performance level $\gamma=2$, the sampling period $h=0.01$, $\alpha=0.5$ in the ETM, switching parameters $\lambda_0=\lambda_2=1.6$, $\eta_1=0.5$, $\eta_2=0.05$, saturation parameter $\sigma=0.1$. Solving the proposed conditions in Theorem 2, one can obtain the weight matrices of the proposed ETM and the filter gains as follows

$$A_{f1}=\begin{bmatrix}-6.0656 & 1.2895 & 1.6369\\ 1.5338 & -4.4588 & 1.5435\\ 0.1667 & -0.0687 & -3.5064\end{bmatrix},$$
$$A_{f2}=\begin{bmatrix}-3.1141 & 0.5180 & 0.3658\\ 0.0644 & -3.6811 & 1.0334\\ 0.6807 & 2.2247 & -2.8927\end{bmatrix},$$
$$B_{f1}=\begin{bmatrix}-0.0456\\ 0.0075\\ -0.0198\end{bmatrix},\ B_{f2}=\begin{bmatrix}0.0234\\ -0.0030\\ -0.0077\end{bmatrix},$$
$$C_{f1}=\begin{bmatrix}-0.5877 & 0.0011 & -0.3126\end{bmatrix},$$
$$C_{f2}=\begin{bmatrix}0.2884 & -0.3260 & -0.2564\end{bmatrix},$$
$$\Omega=6.8291.$$

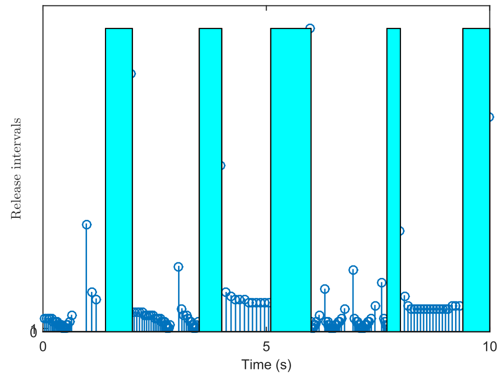


FIGURE 2. **The release instants and their corresponding intervals under hybrid attacks.**

The initial state is assumed as $x(0)=\begin{bmatrix}-0.1 & -0.1 & 0.1\end{bmatrix}^T$. To demonstrate the advantage of our proposed ETM, for this example, we assume the random attack signal is injected from 2s to 3.5s with $\bar{\beta}=0.2$ in (15), and the disturbance occurs from 6s to 8s with the following format

$$\omega(t)=\begin{cases}0.005sin5t, & t\in[6,8)\\ 0, & \text{others}\end{cases}$$

Figure 2 shows the data-releasing sequence and DoS active attack periods. Only 17% sampled data are released into the network due to the impact of the proposed ETM, by which a lot of communication resource can be saved. Although a same result to the ETM and DoS attack can be led, that is, some sampling data can not reach to the filter side, it needs to noted that the main purpose of the DoS attack is to deteriorate the estimation performance, while the purpose of the ETM is to balance performance between the filter and the network by choose necessary sampling data to the filter. Figure 3 and Figure 4 show the output signals $z_f(t)$ and $z(t)$ and the filtering error $e_f(t)$, respectively, from which one can conclude that the filter can maintain the estimation performance with a certain level when the system is subjected to hybrid-attacks and sensor saturation. From Figure 2, one can notice that the average data releasing rate during the system with deception attack from 2s to 3.5s and during the disturbance period from

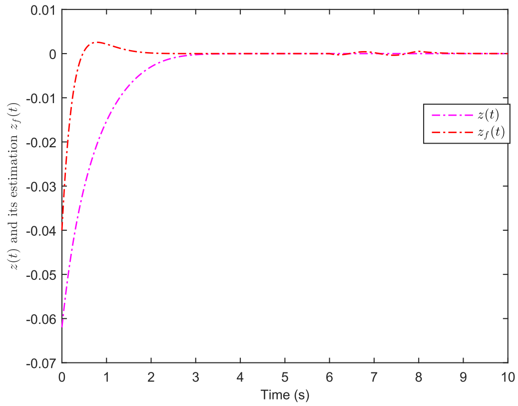


FIGURE 3. Responses of $z(t)$ and $z_f(t)$.

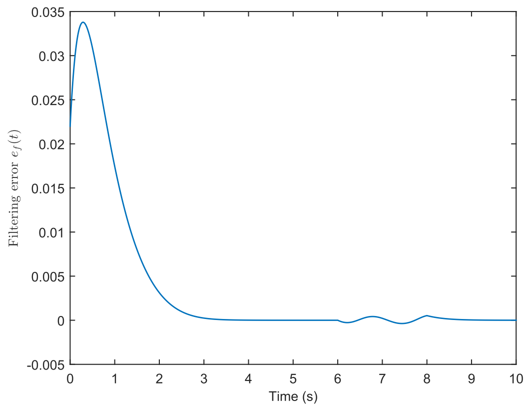


FIGURE 4. The filtering error.

6s to 8s are 35% and 26%, respectively, which is higher than the one without deception attack during the period from 4s to 5.1s. It illustrates that the proposed ETM is sensitive to this type of attack as stated in Remark 4, and is also sensitive to the disturbance. In fact, the deception attack can be regarded as a special external disturbance. Thanks to the proposed ETM, much more data can reach to the filter side when the system is suffered from deception attacks, which results in a good estimation performance of the filter.

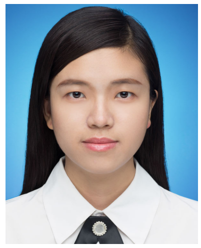
V. CONCLUSION

This paper has been investigated the event-based filtering problem for T-S fuzzy-based nonlinear systems subject hybrid attacks. The hybrid attacks include deception attacks and DoS attacks. A novel ETM is adopted for releasing much more data compared when the system suffers from deception attacks compared to traditional ETM. Moreover, such an ETM can reduce erroneous triggering events aroused from the abrupt output measurement by using a method of average value to the ETM input. In addition, the sensor saturation is also considered in filter design which is a common phenomenon in measurement device. Sufficient conditions and the maximum duration time of DoS active period are derived to ensure the stability of the system under the hybrid attacks. Finally, a numerical example is given to manifest the effectiveness of the design method. It is noted that asymptotically stable results have been obtained in this study, finite-time stable results can be got as well by using the approach like in [33], [34]. Furthermore, it is very important for the researches to apply these theoretical results on the real-world applications, which is one of our main future directions.

REFERENCES

[1] D. Ye, N.-N. Diao, and X.-G. Zhao, "Fault-tolerant controller design for general Polynomial-Fuzzy-Model-Based systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 2, pp. 1046–1051, Apr. 2018.
[2] E. Tian, Z. Wang, L. Zou, and D. Yue, "Chance-constrained H_∞ control for a class of time-varying systems with stochastic nonlinearities: The finite-horizon case," *Automatica*, vol. 107, pp. 296–305, Sep. 2019.
[3] S. Mobayen, "Chaos synchronization of uncertain chaotic systems using composite nonlinear feedback based integral sliding mode control," *ISA Trans.*, vol. 77, pp. 100–111, Jun. 2018.
[4] J. Wang, S. Ma, C. Zhang, and M. Fu, "Finite-time H_∞ filtering for nonlinear singular systems with nonhomogeneous Markov jumps," *IEEE Trans. Cybern.*, vol. 49, no. 6, pp. 2133–2143, Jun. 2019.
[5] Y. Liu, B.-Z. Guo, J. H. Park, and S. Lee, "Event-based reliable dissipative filtering for T–S fuzzy systems with asynchronous constraints," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 4, pp. 2089–2098, Aug. 2018.
[6] S. Zhang, D. Ding, G. Wei, J. Mao, Y. Liu, and F. E. Alsaadi, "Design and analysis of H_∞ filter for a class of T-S fuzzy system with redundant channels and multiplicative noises," *Neurocomputing*, vol. 260, pp. 257–264, Oct. 2017.
[7] H. Du and N. Zhang, "Fuzzy control for nonlinear uncertain electrohydraulic active suspensions with input constraint," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 2, pp. 343–356, Apr. 2009.
[8] O. M. Kwon, M. J. Park, J. H. Park, and S. M. Lee, "Stability and stabilization of T-S fuzzy systems with time-varying delays via augmented Lyapunov-Krasovskii functionals," *Inf. Sci.*, vol. 372, pp. 1–15, Dec. 2016.
[9] L. Ding, Q.-L. Han, X. Ge, and X.-M. Zhang, "An overview of recent advances in event-triggered consensus of multiagent systems," *IEEE Trans. Cybern.*, vol. 48, no. 4, pp. 1110–1123, Apr. 2018.
[10] S. Yan, S. K. Nguang, M. Shen, and G. Zhang, "Event-triggered H_∞ control of networked control systems with distributed transmission delay," *IEEE Trans. Autom. Control*, early access, Nov. 14, 2019, doi: 10.1109/TAC.2019.2953460.
[11] K. Wang, E. Tian, J. Liu, L. Wei, and D. Yue, "Resilient control of networked control systems under deception attacks: A memory-event-triggered communication scheme," *Int. J. Robust Nonlinear Control*, vol. 30, no. 4, pp. 1534–1548, Mar. 2020.
[12] Y. Yang, D. Yue, and C. Dou, "Output-based event-triggered schemes on leader-following consensus of a class of multi-agent systems with Lipschitz-type dynamics," *Inf. Sci.*, vol. 459, pp. 327–340, Aug. 2018.
[13] J. Liu, L. Wei, X. Xie, E. Tian, and S. Fei, "Quantized stabilization for T–S fuzzy systems with hybrid-triggered mechanism and stochastic cyber-attacks," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3820–3834, Dec. 2018.
[14] Y.-X. Li and G.-H. Yang, "Model-based adaptive event-triggered control of strict-feedback nonlinear systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 4, pp. 1033–1045, Apr. 2018.
[15] Z. Gu, P. Shi, D. Yue, and Z. Ding, "Decentralized adaptive event-triggered H_∞ filtering for a class of networked nonlinear interconnected systems," *IEEE Trans. Cybern.*, vol. 49, no. 5, pp. 1570–1579, May 2019.
[16] S. Yan, S. K. Nguang, and L. Zhang, "Nonfragile integral-based event-triggered control of uncertain cyber-physical systems under cyber–attacks," *Complexity*, vol. 2019, pp. 1–14, Nov. 2019, doi: 10.1155/2019/8194606.
[17] Z. Gu, X. Zhou, T. Zhang, F. Yang, and M. Shen, "Event-triggered filter design for nonlinear cyber–physical systems subject to deception attacks," *ISA Trans.*, early access, Mar. 14, 2019, doi: 10.1016/j.isatra.2019.02.036.
[18] H. Sun, C. Peng, T. Yang, H. Zhang, and W. He, "Resilient control of networked control systems with stochastic denial of service attacks," *Neurocomputing*, vol. 270, pp. 170–177, Dec. 2017.
[19] S. Hu, D. Yue, X. Xie, X. Chen, and X. Yin, "Resilient event-triggered controller synthesis of networked control systems under periodic DoS jamming attacks," *IEEE Trans. Cybern.*, vol. 49, no. 12, pp. 4271–4281, Dec. 2019.

[20] D. Ye and T.-Y. Zhang, "Summation detector for false data-injection attack in cyber-physical systems," *IEEE Trans. Cybern.*, vol. 50, no. 6, pp. 2338–2345, Jun. 2020, doi: 10.1109/TCYB.2019.2915124.
[21] D. Ding, Z. Wang, D. W. C. Ho, and G. Wei, "Distributed recursive filtering for stochastic systems under uniform quantizations and deception attacks through sensor networks," *Automatica*, vol. 78, pp. 231–240, Apr. 2017.
[22] E. Tian and C. Peng, "Memory-based event-triggering H_∞ load frequency control for power systems under deception attacks," *IEEE Trans. Cybern.*, early access, Mar. 12, 2020, doi: 10.1109/TCYB.2020.2972384.
[23] Z. Gu, J. H. Park, D. Yue, Z.-G. Wu, and X. Xie, "Event-triggered security output feedback control for networked interconnected systems subject to cyber-attacks," *IEEE Trans. Syst., Man, Cybern., Syst.*, early access, Jan. 10, 2020, doi: 10.1109/TSMC.2019.2960115.
[24] P.-P. Wang and W.-W. Che, "Quantized H_∞ filter design for networked control systems with random nonlinearity and sensor saturation," *Neurocomputing*, vol. 193, pp. 14–19, Jan. 2016.
[25] Y. Zhu, L. Zhang, and M. V. Basin, "Nonstationary H_∞ dynamic output feedback control for discrete-time Markov jump linear systems with actuator and sensor saturations," *Int. J. Robust Nonlinear Control*, vol. 26, no. 5, pp. 1010–1025, Mar. 2016.
[26] H. Ji, H. Zhang, C. Li, S. Tian, and Y. Wei, "H_∞ control for time-delay systems with randomly occurring nonlinearities subject to sensor saturations, missing measurements and channel fadings," *ISA Trans.*, vol. 75, pp. 38–51, Apr. 2018.
[27] J. Liu, Y. Gu, J. Cao, and S. Fei, "Distributed event-triggered H_∞ filtering over sensor networks with sensor saturations and cyber-attacks," *ISA Trans.*, vol. 81, pp. 63–75, Oct. 2018.
[28] D. Yue, E. Tian, and Q.-L. Han, "A delay system method for designing event-triggered controllers of networked control systems," *IEEE Trans. Autom. Control*, vol. 58, no. 2, pp. 475–481, Feb. 2013.
[29] H. Ren, G. Zong, and H. R. Karimi, "Asynchronous finite-time filtering of networked switched systems and its application: An event-driven method," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 66, no. 1, pp. 391–402, Jan. 2019.
[30] S. Mobayen, "Design of LMI-based sliding mode controller with an exponential policy for a class of underactuated systems," *Complex.*, vol. 21, no. 5, pp. 117–124, 2016.
[31] D. Zhang, Q.-L. Han, and X. Jia, "Network-based output tracking control for T–S fuzzy systems using an event-triggered communication scheme," *Fuzzy Sets Syst.*, vol. 273, pp. 26–48, Aug. 2015.
[32] H. Wang, P. Shi, and J. Zhang, "Event-triggered fuzzy filtering for a class of nonlinear networked control systems," *Signal Process.*, vol. 113, pp. 159–168, Aug. 2015.
[33] H. Ren, G. Zong, and T. Li, "Event-triggered finite-time control for networked switched linear systems with asynchronous switching," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 11, pp. 1874–1884, Nov. 2018.
[34] F. Bayat, S. Mobayen, and S. Javadi, "Finite-time tracking control of N th-order chained-form non-holonomic systems in the presence of disturbances," *ISA Trans.*, vol. 63, pp. 78–83, Jul. 2016.



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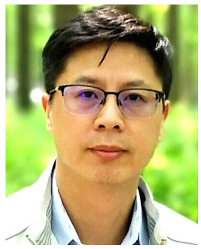

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